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APPLICATION OF THE SECOND ASYMPTOTIC APPROXIMATION OF PULSE DISTORTION IN ACOUSTIC DIAGNOSTICS OF A LAYER ON A HALF-SPACE

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One-dimensional echo reflected from the plane interface of two media in which the wave pattern is defined by quasilinear wave equations is considered. Depending on the physical meaning of coefficients of the equation, the pattern of nonlinear wave distortion propagation is defined either in an elastic or a perfectly compressible fluid [1]. The second asymptotic approximation is derived by the method of successive integration of inhomogeneous wave equations, similar to that used in [1 — 4] for the first asymptotic approximation.

1. Let us consider the one-dimensional wave pattern in a medium consisting of layer A on half-space B (Fig. 1). We use the following notation: t for time, X for the Lagrangin coordinate; U_j, σ_{11j}, and ρ_j $(j = A, B)$, respectively, for the lengthwise displacement, the longitudinal component of the Lagrange pseudostress tensor, and the densities of layer A and half-space B in their initial state. The prime and the dot denote derivatives with respect to X and t, respectively.

We define the wave pattern by the equations

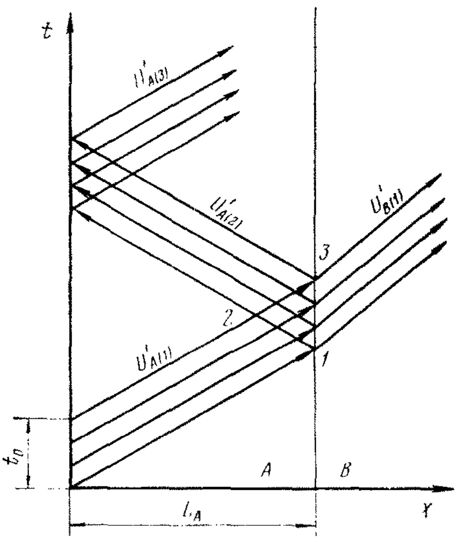

Fig. 1

$$\sigma_{11j}{}' (X, t) = \rho_j U_j{}^{\cdot\cdot} (X, t), \quad j = A, B \tag{1.1}$$

$$\sigma_{11j} (X, t) \equiv Q_j (U_l{}') = P_0 + \beta_j [U'_j + {}^1\!/_2 k_{1j} \times (U_j{}')^2 + {}^1\!/_3 k_{2j} (U_i{}')^3 + \ldots]$$

where P_0, β_j, k_{ij} are constant coefficients, and introduce the definitions

$$q_j (U_j{}') = \beta_j^{-1}\, dQ_j (U_j{}')/dU_j{}', \quad c_j{}^2 = \beta_j \rho_j{}^{-1}$$

From (1.1) we obtain the equation

$$c_j^{-2} U_j{}^{\cdot\cdot} (X, t) - q_j (U_j{}') U_j{}'' (X, t) = 0, \quad j = A, B \tag{1.2}$$

We assume that the condition

$$
\begin{aligned}
&U_j(X, 0) = 0, \quad U_j^{\cdot}(X, 0) = 0 \\
&U_A^{\cdot}(0, t) = -\varepsilon c_A \psi(t)\,[H(t) - H(t - t_0)], \\
&|\,\varepsilon\,| \ll 1;\ 0 \leqslant t_0 < c_A^{-1} L_A \\
&U_A^{\cdot}(L_A, t) = U_B^{\cdot}(L_A, t),\ \sigma_{11A}(L_A, t) = \sigma_{11B}(L_A, t)
\end{aligned}
\tag{1.3}
$$

are satisfied as the initial conditions, the boundary condition at boundary $X = 0$, and the condition of contact at the interface $X = L_A$, respectively. In these formulas $H(t)$ is the Heaviside function and $\psi(t)$ an arbitrary continuous function that satisfies conditions

$$
\begin{aligned}
&\psi(0) = \psi(t_0) = 0, \quad \psi^{\cdot}(0) = \psi^{\cdot}(t_0) = 0 \\
&\max\ |\,\psi(t)\,| = 1, \quad 0 < t < t_0
\end{aligned}
$$

and in the interval $0 \leqslant t \leqslant t_0$ has continuous derivatives of all orders required subsequently.

The boundary effect at $X = 0$ (the third of conditions (1.3)) generates a finite length pulse which we denote by $U_{A(1)}^{\cdot}(X, t)$, whose reflection and refraction at the two media interface $X = L_A$ produce the reflected pulse $U_{A(2)}^{\cdot}(X, t)$ and the refracted pulse $U_{B(1)}^{\cdot}(X, t)$ (Fig. 1). The reflected pulse $U_{A(2)}^{\cdot}(X, t)$ is the echo from the interface $X = L_A$.

Using the method expounded in [1, 3] we obtain with an accuracy to the second asymptotic approximation the following definition of the echo:

$$
\begin{aligned}
&U_{A(2)2}^{'}(X, t) = [H(t_{2A}) - H(t_{2A} - t_0)]\Big\{\sum_{i=0}^{10} J_i F_i + \\
&[H(t_{1A}) - H(t_{1A} - t_0)] \sum_{i=11}^{33} J_i F_i\Big\} \\
&t_{1A} = t - c_A^{-1} X, \quad t_{2A} = t - 2c_A^{-1} L_A + c_A^{-1} X
\end{aligned}
\tag{1.4}
$$

where $F_i = F_i(X, t)$ are functions determined by the specification of function $\psi(t)$ appearing in the boundary condition at $X = 0$, and the quantities $J_i = J_i(X)$ depend on the constants of the considered medium

$$
\begin{aligned}
&F_0 = \psi_2, \quad F_1 = F_0^2, \quad F_2 = F_0^{\cdot}, \quad F_3 = F_1^{\cdot}, \quad F_4 = M_2 F_2 \\
&F_5 = \psi_2^3, \quad F_6 = F_5^{\cdot}, \quad F_7 = F_5^{\cdot\cdot}, \quad F_8 = N_2 F_2, \quad F_9 = M_2 F_3 \\
&F_{10} = M_2^2 \psi_2^{\cdot\cdot}, \quad F_{11} = \psi_1 \psi_2, \quad F_{12} = M \psi_1^{\cdot}, \quad F_{13} = (M_1 - M_0)\,\psi_2^{\cdot} \\
&F_{14} = F_1^{\cdot}, \quad F_{15} = M_1 \psi_2^{\cdot}, \quad F_{16} = \psi_1 \psi_2^2, \quad F_{17} = \psi_1^2 \psi_2 \\
&F_{18} = \psi_1^{\cdot} \psi_2^2, \quad F_{19} = \psi_1 F_3, \quad F_{20} = \psi_1^2 \psi_2^{\cdot}, \quad F_{21} = (\psi_1^2)^{\cdot} \psi_2 \\
&F_{22} = N_2 \psi_1^{\cdot}, \quad F_{23} = N_1 \psi_2^{\cdot}, \quad F_{24} = M_2 (\psi_1^2)^{\cdot}, \quad F_{25} = M_1 F_3 \\
&F_{26} = M_1^2 \psi_2^{\cdot\cdot}, \quad F_{27} = M_2^2 \psi_1^{\cdot\cdot}, \quad F_{28} = M_2 (\psi_1^2)^{\cdot\cdot}, \quad F_{29} = M_1 F_3^{\cdot} \\
&F_{30} = M_1 \psi_1 \psi_2^{\cdot}, \quad F_{31} = M_2 \psi_1 \psi_2^{\cdot}, \quad F_{32} = M_1 \psi_1^{\cdot} \psi_2, \quad F_{33} = M_2 \psi_1^{\cdot} \psi_2 \\
&\psi_k = \psi(t_{kA}), \quad \psi_k^{\cdot} = \psi^{\cdot}(t_{kA}), \quad M_k = \int_0^{t_{kA}} \psi(z)\,dz
\end{aligned}
\tag{1.5}
$$

$$M_0 = \int_0^{t_0} \psi(z)\,dz, \quad N_k = \int_0^{t_{kA}} \psi^2(z)\,dz, \quad k = 1, 2$$

$$J_0 = \varepsilon J_A \tag{1.6}$$

$$J_1 = \frac{1}{16}\varepsilon^2 k_{1A}[2(J_{1A}J_{2A}^2K_A - 2J_A{}^2) + \varepsilon k_{1A}J_A{}^3]$$

$$J_2 = \frac{1}{4}\varepsilon^2 k_{1A}J_A M_0$$

$$J_3 = \frac{1}{16}\varepsilon^2 k_{1A}J_A[4c_A^{-1}(L_AJ_{2A} - J_AX) - \varepsilon k_{1A}J_AM_0]$$

$$J_4 = -\frac{1}{16}J_A[4\varepsilon^2 k_{1A}J_{1A} - \varepsilon^3(3k_{1A}^2 - 4k_{2A})]$$

$$J_5 = \frac{1}{96}\varepsilon^3 k_{1A}^2[2(5 + 7J_A) - 12J_A{}^2(1 + 2J_A - J_A{}^3) -$$
$$J_{1A}^3J_{2A}(3J_A + 7)K_B{}^2 + 3J_{1A}^2J_{2A}^2(3J_A - 1)K_B] -$$
$$\frac{1}{6}\varepsilon^3 k_{2A}J_A(J_A{}^2 + J_{2A}^3) + \frac{1}{6}\varepsilon^3 k_{2B}J_AJ_{1A}^3J_{2A}c_A{}^2c_B^{-2}$$

$$J_6 = \frac{1}{6}\varepsilon^3 c_A^{-1}\left\{\frac{1}{8}k_{1A}^2\{12J_A{}^3X + L_A[2J_{1A}^2J_{2A}K_B - 2J_{1A}^2J_{2A}^2 -\right.$$
$$\left.J_A(13 + 11J_A{}^2)]\} + k_{2A}J_A[L_A(J_A{}^2 + 1) - J_A{}^2X]\right\}$$

$$J_7 = \frac{1}{24}\varepsilon^3 k_{1A}^2c_A^{-2}[J_A{}^3(5L_A{}^2 - 4L_AX - X^2) + J_AL_A{}^2]$$

$$J_8 = \frac{1}{4}\varepsilon^3 J_A\left\{\frac{1}{8}k_{1A}^2[5J_A(2 - J_A) - 1] + k_{2A}J_A{}^2\right\}$$

$$J_9 = -\frac{1}{64}\varepsilon^3 J_Ak_{1A}^2J_{1A}J_{2A}(3J_A - 2J_{1A}K_B)$$

$$J_{10} = -\frac{1}{16}\varepsilon^3 k_{1A}^2J_A{}^2J_{1A}, \quad J_{11} = -\frac{1}{16}J_A(8\varepsilon^2 k_{1A} - \varepsilon^3 k_{1A}^2J_A)$$

$$J_{12} = J_{13} = \frac{1}{4}\varepsilon^2 k_{1A}J_A, \quad J_{14} = \frac{1}{16}\varepsilon^3 k_{1A}^2J_A{}^2M_0$$

$$J_{15} = -\frac{1}{4}\varepsilon^3 J_A\left(\frac{3}{4}k_{1A}^2 - k_{2A}\right), \quad J_{16} = \frac{1}{2}\varepsilon^3 J_A{}^2\left(\frac{5}{8}k_{1A}^2 - k_{2A}\right)$$

$$J_{17} = \frac{1}{2}\varepsilon^3 J_A(k_{1A}^2 - k_{2A}),$$

$$J_{18} = -\frac{1}{2}J_{19} = \frac{1}{16}\varepsilon^3 k_{1A}^2c_A^{-1}J_A{}^2(2L_A - X)$$

$$J_{20} = -\frac{1}{2}J_{21} = \frac{1}{16}\varepsilon^3 k_{1A}^2c_A^{-1}XJ_A, \quad J_{22} = -\frac{1}{2}J_{16},$$

$$J_{23} = \frac{1}{32}\varepsilon^3 k_{1A}^2J_A$$

$$J_{24} = J_A^{-1}J_{25} = -\frac{1}{4}\varepsilon^3 J_A\left(\frac{9}{8}k_{1A}^2 - k_{2A}\right), \quad J_{26} = J_A^{-1}J_{27} = J_{23}$$

$$J_{28} = J_{20}, \quad J_{29} = J_{18},$$

$$J_{30} = -2J_A^{-1}J_{31} = -2J_{32} = J_A^{-1}J_{33} = -\frac{1}{8}\varepsilon^3 k_{1A}^2J_A$$

$$J_A = \frac{\rho_Bc_B - \rho_Ac_A}{\rho_Bc_B + \rho_Ac_A}, \quad J_{1A} = 1 - J_A, \quad J_{2A} = 1 + J_A \tag{1.7}$$

$$K_A = \frac{k_{1B}\beta_A}{k_{1A}\beta_B} - 1, \quad K_B = \frac{k_{1B}c_A}{k_{1A}c_B}$$

Note that in conformity with (1.4) the reflected pulse is defined in the nonlinear interaction region of the incident and reflected pulses (see triangle 123 in Fig. 1) by the totality of all terms $(i = 0, 1, \ldots, 33)$ of that formula, while outside that region it is defined by the first sum in braces $(i = 0, 1, \ldots, 10)$ of the same formula. The term with $i = 0$ which contains the first power of ε determines the linear (zero) approximation, while terms containing the second and third powers of ε determine corrections for the nonlinear effects within the accuracy of the first and second asymptotic approximations, respectively. The terms $i = 0, 1, \ldots, 4$ and $i = 11, 12, 13$ in the sum in (1.4) are accurate within the first asymptotic approximation derived in [1,3] with coefficients J_i determined within the accuracy of terms containing ε in the first and second powers.

2. Let us consider some fixed point $X = a$ outside the region of interaction of pulses $U'_{A(1)}(X, t)$ or $U'_{A(2)}(X, t)$ in which the echo is determined by the first sum in braces in formula (1.4). We assume that at that point the echo is registered and its time lag

$$r = 2c_A^{-1}L_A - c_A^{-1}a \tag{2.1}$$

measured.

We assume that function $\psi(t)$ can be chosen so that the echo $U'_{A(2)}(X, t)$ can be decomposed in components whose time dependence is defined by functions $F_i(a, t)$, of corresponding amplitudes J_i. Then the coefficients J_i can be assumed to be the experimentally obtained constants for $X = a$.

Let us consider the information on parameters of layer A and half-space B that can be extracted from the numerical values of $r, J_0, J_1, J_2, \ldots$. The [boundary] effect amplitude $\varepsilon_1 \equiv \varepsilon c_A$ and the value of the definite integral M_0 are taken to be specified constants of which the last vanishes when these effects are "balanced".

Actual acoustic measurements are, evidently, obtained with some known accuracy, hence we can only consider those of constants $J_1, J_2, \ldots,$ that are not too small in comparison with J_0 and the noise level, and allow for some error in the determination of each of constants $r, J_0, J_1, J_2, \ldots$. Below, we consider the following groups of experimentally determined constants.

1°. Parameters r and J_0 of the echo linear component that are expressed in terms of the [boundary] effect amplitude $\varepsilon_1 = \varepsilon C_A$ and of parameters of layer A and half-space B in conformity with formulas (1.6) and (2.1).

2°. Parameters $J_1^\circ, J_2, J_3^\circ$ and J_4°, of the echo first order nonlinear components that are expressed in terms of parameters of layer A and half-space B in conformity with formulas (1.6), with J_1°, J_3°, and J_4° denoting the respective principal parts of J_1, J_3, and J_4 containing ε^2.

3°. Parameters of the echo second order nonlinear components J_j $(j = 1, 3, 4, \ldots, 10)$ that are determined in conformity with formulas (1.6).

Problems of information extraction from parameters of groups 1° and 2° were considered in [1 — 3] for various forms of boundary effects and measurable quantities. The possibility of information extraction from parameters of group 3° can be analyzed in a similar manner. We shall formulate the basic results without going into the details of the analysis.

We assume that the [boundary] effect amplitude $\varepsilon_1 = \varepsilon c_A$ and the function $\psi(t)$ of that effect variation — and when $M_0 = 0$, also, the distance from point a of interaction to the point of echo measurement — are known.

If only values of constants r and J_0 of group 1° are obtained from experimental data, it is possible to calculate the quantities

$$(2L_A - a) / c_A, \quad J_A / c_A \tag{2.2}$$

If values of constants of groups 1° and 2° are available from such data, it is possible to calculate the quantities

$$k_{1A}, \ k_{1B} / c_B \tag{2.3}$$

and when $M_0 \neq 0$, $\varepsilon, c_A, L_A, J_A$

If one succeeds in extracting from experimental data the values of groups 1° — 3°, it becomes possible to calculate in addition to (2.3) the quantities

$$k_{2A}, \quad k_{2B} / c_B^2 \tag{2.4}$$

while simultaneously increasing the accuracy of calculation of quantities (2.3).

These conclusions are based on relationships

$$\frac{\rho_B c_B}{\rho_A c_A} = \frac{J_{2A}}{J_{1A}}, \qquad \frac{k_{1B}}{c_B} = \frac{1 + K_A}{c_A} k_{1A} \frac{\rho_B c_B}{\rho_A c_A} = \frac{K_B k_{1A}}{c_A} \tag{2.5}$$

obtained with the use of definitions of c_A and c_B, and formulas (1.7).

A p a r t i c u l a r c a s e. When the layer A and the half-space B are perfect incompressible fluids we have [1]

$$k_{1i} = -(\gamma_i + 1), \quad k_{2i} = {}^1/_2 (\gamma_i + 1)(\gamma_i + 2), \quad i = A, B \tag{2.6}$$

where γ_A and γ_B are the adiabatic exponents of fluids in layer A and half-space B, respectively. In this case it is possible to determine, using the experimentally determined quantities k_{1B} / c_B and k_{2B} / c_B^2, the ratio k_{1B}^2 / k_{2B} which in accordance with formulas (2.6) is expressed in terms of γ_B as follows:

$$\frac{k_{1B}^2}{k_{2B}} = \frac{\gamma_B^2 + 2\gamma_B + 1}{{}^1/_2\gamma_B^2 + {}^3/_2\gamma_B + 1} \tag{2.7}$$

from which follows

$$\gamma_B = -\frac{2(R-1)}{R-2}, \quad R = \frac{k_{1B}^2}{k_{2B}} \tag{2.8}$$

Having obtained the numerical value of γ_B, we can calculate k_{1B} and k_{2B}, and then c_B. Since $\beta_B c_B / \rho_A c_A$ and c_A are now known quantities, it is not difficult to calculate the ratio ρ_B / ρ_A.

3. In applications it may prove to be more advantageous, or necessary, not to decompose the reflected pulse in components whose time dependence is determined by respective functions $F_i(a, t)$, but in some other manner.

For instance, when a source is generating sinusoidal pulses

$$U_A^{\bullet}(0, t) = -\varepsilon c_A [H(t) - H(t - t_0)] \sin \omega t \tag{3.1}$$

the echo consists of components (harmonics) of frequencies $\omega, 2\omega, 3\omega, \ldots$. Amplitudes of the first three harmonics are obtained from the solution of the inverse problem of acoustic diagnosis using second approximation formulas.

The value of information provided by the echo can be increased by using the source of the so-called "parametric" pulses of the form

$$U_A^{\bullet}(0, t) = -\varepsilon c_A [H(t) - H(t - t_0)] \psi(t) \tag{3.2}$$

$$\psi(t) = {}^1/_2 (\cos 2\omega_2 t - \cos 2\omega_1 t), \quad \omega_2 > \omega_1 \tag{3.3}$$

With these and the use of second approximation formulas for solving the inverse problem of acoustic diagnostics, we obtain echo components of frequencies $2\omega_1$, $4\omega_1$, $6\omega_1$, $2\omega_2$, $4\omega_2$, $6\omega_2$, $2(\omega_2 \pm \omega_1)$, $2(2\omega_2 \pm \omega_1)$ and $2(\omega_2 \pm 2\omega_1)$. The number of different frequency components whose amplitude contains information is, thus, increased from three in the first case to twelve in the second. The echo constant component is not considered here.

Let us consider again the signal outside the region of its interaction with the fading pulse. The substitution of (3.2) and (3.3) into (1.4) yields

$$U'_{A(2)2}(a, t) = [H(t_{2A}) - H(t_{2A} - t_0)] \times \tag{3.4}$$

$$\left[a_0 + \sum_{i=1}^{12} (a_i \cos \Omega_i t_{2A} + \Omega_i b_i \sin \Omega_i t_{2A})\right]$$

$$\Omega_i = 2i\omega_1, \quad \text{if} \quad i = 1, 2, 3$$

$$\Omega_i = 2(i - 3)\omega_2, \quad \text{if} \quad i = 4, 5, 6$$

$$\Omega_i = 2[\omega_1 + (-1)^i \omega_2], \quad \text{if} \quad i = 7, 8$$

$$\Omega_i = 2[\omega_1 + (-1)^i 2\omega_2], \quad \text{if} \quad i = 9, 10$$

$$\Omega_i = 2[2\omega_1 + (-1)^i \omega_2], \quad \text{if} \quad i = 11, 12$$

$$a_0 = \frac{1}{4}(J_1 - J_4) \tag{3.5}$$

$$a_1 = -\frac{1}{64}\left[32J_0 + 18J_5 - 72\omega_1^2 J_7 - \frac{9\omega_2^2 - \omega_1^2}{\omega_2^2 - \omega_1^2} J_8 - 12J_9 - 2\left(1 + \frac{2\omega_1^2}{\omega_2^2}\right) J_{10}\right]$$

$$a_2 = a_5 = \frac{1}{8}(J_1 + J_4)$$

$$a_3 = -\frac{1}{64}(2J_5 - 72\omega_1^2 J_7 + J_8 + 4J_9 + 2J_{10})$$

$$a_4 = \frac{1}{64}\left[32J_0 + 18J_5 - 72\omega_2^2 J_7 + \frac{9\omega_1^2 - \omega_2^2}{\omega_2^2 - \omega_1^2} J_8 - 12J_9 - 2\left(1 + \frac{2\omega_2^2}{\omega_1^2} J_{10}\right)\right]$$

$$a_6 = \frac{1}{64}(2J_5 - 72\omega_2^2 J_7 + J_8 + 4J_9 + 2J_{10})$$

$$a_{7,8} = -\frac{1}{8}\left[2J_1 \mp \left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1}\right) J_4\right]$$

$$a_{9,10} = \frac{1}{64}\left[-6J_5 + 24(\omega_1 \mp 2\omega_2)^2 J_7 - \frac{\omega_1\omega_2 \mp \omega_1^2 \mp 4\omega_2^2}{\omega_2(\omega_1 \mp \omega_2)} J_8 - 4\left(1 \mp \frac{\omega_1}{\omega_2} \mp \frac{\omega_2}{\omega_1}\right) J_9 - 4\left(\frac{\omega_1^2}{2\omega_2^2} \mp \frac{\omega_2}{\omega_1}\right) J_{10}\right]$$

$$a_{11,12} = \frac{1}{64}\left[6J_5 - 24(\omega_2 \mp 2\omega_1)^2 J_7 + \frac{\omega_1\omega_2 \mp \omega_2^2 \mp 4\omega_1^2}{\omega_1(\omega_2 \mp \omega_1)} J_8' + 4\left(1 \mp \frac{\omega_1}{\omega_2} \mp \frac{\omega_2}{\omega_1}\right) J_9 + 4\left(\frac{\omega_2^2}{2\omega_1^2} \mp \frac{\omega_1}{\omega_2}\right) J_{10}\right]$$

$$b_1 = -b_4 = {}^1/_{32}(16J_2 + 9J_6 + 4t_{2A}J_8)$$

$$b_2 = b_5 = -\frac{J_3}{8}, \quad b_3 = -b_6 = \frac{J_6}{32}, \quad b_7 = b_8 = \frac{J_3}{4}$$

$$b_9 = b_{10} = \frac{3J_6}{32}, \quad b_{11} = b_{12} = -\frac{3J_6}{32}$$

Let us assume that the processing of experimental data has provided numerical values of amplitudes a_i and $\Omega_i b_i$ of various components of the echo (3.4). The extraction of information from the echo then reduces to the determination of quantities $J_i\,(i = 0, 1, 2, \ldots)$ on the basis of a_i and $b_i\,(i = 1, 2, \ldots)$, using formulas (3.5), since the information extraction from $J_i\,(i = 0, 1, 2, \ldots)$ has been already dealt with in a general manner in Sect. 2. Since each of coefficients a_i and b_i depends on several J_i, it is important, when calculating the latter by formulas (3.5), to evaluate the order of magnitude of terms in these formulas.

Let us consider the case

$$\omega_1 \sim \omega_2 \sim \Omega,\ (\omega_2 - \omega_1)/\omega_2 \sim \eta,\ 0 < \eta \ll 1$$
$$I_A \sim 1,\ K_A \sim 1,\ K_B \sim 1,\ \Omega M_0 \sim 1$$

From initial assumptions and definitions we have

$$|\varepsilon k_{1A}| \ll 1,\ \max t_{2A} = t_0,\ \Omega = n\pi t_0^{-1} \tag{3.6}$$

We introduce constant α which we define as follows:

$$L_A n\pi / (t_0 c_A) \sim (\varepsilon k_{1A})^{-\alpha} \tag{3.7}$$

On the basis of formulas (1.6), (3.4), and (3.5) we can conclude that application of the asymptotic method used here is justified only in the case of problem parameters that in (3.7) correspond to $\alpha < 1$. We admit that this condition is fulfilled. Then, taking into account (3.6) and (3.7), we obtain from formulas (1.6) and (3.5) the following estimates for J_j :

$$J_0 \sim \varepsilon;\ J_1,\ J_2,\ J_4 \sim \varepsilon^2 k_{1A};\ J_3,\ J_6 \sim \varepsilon^2 k_{1A} L_A c_A^{-1}$$
$$J_5,\ J_8,\ J_9,\ J_{10} \sim \varepsilon^3 k_{1A}^2;\ J_7 \sim \varepsilon^3 k_{1A}^2 L_A^2 c_A^{-1}$$

Coefficients $J_j\,(j = 0, 1, \ldots, 6, 9, 10)$ in formulas (3.5) are of order unity, the coefficient at J_7 is of order Ω^2, and at J_8 is of order η^{-1} in the calculation of a_1, a_4, a_9, a_{11} and of order unity in the calculation of a_3, a_6, a_{10} and a_{12}.

Taking the aforesaid into account, we conclude that in this case the calculation of J_j using a_j and b_j may be carried out as follows: J_0 is calculated using a_1 or a_4; J_1 and J_4 using the system of equations for a_2 and a_7; J_2 using b_1 or b_4; J_3 using b_2 or b_5; J_6 using b_3 or b_6, and $J_5, J_7, J_8, J_9, J_{10}$ are calculated using the system of equations for a_8, a_9, a_{10}, a_{11} and a_{12}.

REFERENCES

1. N i g u l, U. K. Echos from Elastic Objects, Vol. 1. Tallin, "Valgus", 1976.

2. N i g u l, U., Asymptotic analysis of the nonlinear effects of finite echo pulses from elastic half-space. Acoustica, Vol. 34, No. 3, 1976.

3. N i g u l, U. K., Asymptotic analysis of the possibility of using nonlinear effects in acoustical diagnostics of the properties of the elastic object "layer on half-space". Mekhanika Polimerov, No. 2, 1977.

4. N i g u l, U. K., Asymptotic determination of the formation process of nonlinear distortion of one-dimensional pulses in a layered medium. PMM, Vol. 40, No. 6, 1976.

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